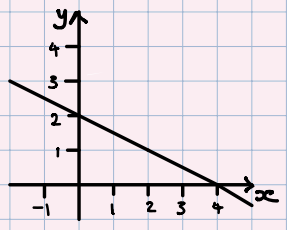
Straight Lines

$$y = mx + c$$

m: gradient $= \frac{\Delta y}{\Delta x}$

c: y-intercept.

Finding the equation of a line from a graph



Gradient, $m = \frac{\Delta y}{\Delta x}$

choose 2 points on the line.
eg. (0,2) and (4,0)

$$m = \frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$$

y-intercept, c

where the line crosses the y-axis.

$c = 2$.

$$y = mx + c$$

$$y = -\frac{1}{2}x + 2$$

Sometimes the question might ask you to give the equation in the form $ax + by + c = 0$.

This means we need to rearrange our equation.

$$y = -\frac{1}{2}x + 2$$

(move everything to left side of equation)

$$\frac{1}{2}x + y - 2 = 0$$

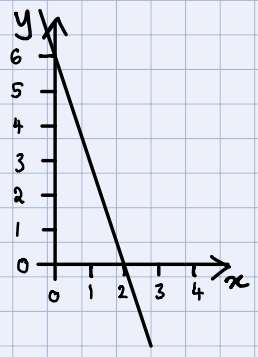
(multiply by 2 to make coefficients whole numbers)

$$x + 2y - 4 = 0$$

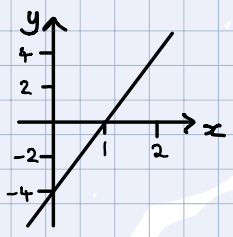
Your turn!

Find the equations of the lines:

a)



b)



Answers

a) $m = \frac{0-6}{2-0} = \frac{-6}{2} = -3$

$c = 6$ $\Rightarrow y = -3x + 6$

b) $m = \frac{4-0}{2-1} = \frac{4}{1} = 4$

$c = -4$ $\Rightarrow y = 4x - 4$

Finding the equation of a line given two points

(7, 2) and (10, -13)

gradient, $m = \frac{-13-2}{10-7} = \frac{-15}{3} = -5$

$y = -5x + c$.

y-intercept, c.

To find c, we substitute either one of the coordinates into our equation.

Let's use (7, 2) or use (10, -13)

$y = -5x + c$ $\quad$ $y = -5x + c$

$2 = -5(7) + c$ $\quad$ $-13 = -5(10) + c$

$2 = -35 + c$ $\quad$ $-13 = -50 + c$

$c = 37$ $\quad$ $c = 37$

$\Rightarrow y = -5x + 37$.

Your turn!

Find the equation of the line;

a) through the points $(-7, 9)$ and $(-3, 6)$

b) with gradient 6, and goes through the point $(0, 5)$.

Answers

a) $m = \frac{6-9}{-3--7} = \frac{-3}{4}$

$y = -\frac{3}{4}x + c$

Sub in $(-7, 9)$ (or $(-3, 6)$)

$9 = -\frac{3}{4}(-7) + c$

$c = \frac{15}{4}$

$\Rightarrow y = -\frac{3}{4}x + \frac{15}{4}$

b) $m = 6$

Sub in $(0, 5)$

$y = 6x + c$

$5 = 6(0) + c$

$c = 5$

$\Rightarrow y = 6x + 5$

Rearranging a line to get the gradient

$y = 7x + 4 \quad \Rightarrow m = 7$

$y = 9 - \frac{x}{5} \quad \Rightarrow y = -\frac{1}{5}x + 6 \quad \Rightarrow m = -\frac{1}{5}$

$2x - 3y = 9 \quad \Rightarrow -3y = -2x + 9 \Rightarrow y = \frac{2}{3}x - 3 \Rightarrow m = \frac{2}{3}$

Your turn!

Find the gradient of the line $4x + 7y - 21 = 0$.

Answer

$4x + 7y - 21 = 0$

$7x + 7y = 21$

$7y = -7x + 21$

$y = -x + 3 \Rightarrow m = -1$

Parallel and Perpendicular lines.

Parallel lines have the same gradient

Find the equation of the line parallel to $y = -\frac{x}{4} + 1$, and goes through the point $(7, -1)$.

$$y = mx + c$$

gradient, m

parallel so the same,

$m = -\frac{1}{4}$

y-intercept, c

$y = -\frac{1}{4}x + c$

substitute in $(7, -1)$

$-1 = -\frac{1}{4}(7) + c$

$c = -1 + \frac{7}{4} = \frac{3}{4}$

$\Rightarrow y = -\frac{1}{4}x + \frac{3}{4}$

Perpendicular lines.

→ Their gradients multiply together to give -1.

$m_1 \times m_2 = -1$.

$\Rightarrow m_2 = \frac{-1}{m_1}$

Find the equation of the line perpendicular to $y = -\frac{x}{4} + 1$, and goes through $(1, -1)$

gradient, m

perpendicular,

$m_2 = \frac{-1}{-\frac{1}{4}} = 4$

y-intercept, c

$y = 4x + c$

substitute in $(1, -1)$

$-1 = 4(1) + c$

$-1 = 4 + c$

$c = -5$

$\Rightarrow y = 4x - 5$

Your turn!

Find the equation of the line;

a) parallel to $3y + \frac{x}{4} + 1 = 0$ and goes through $(0, 3)$

Hint. first, rearrange to $y = \dots\dots$

b) perpendicular to $4x + 2y - 15 = 0$ and goes through $(1, 4)$

Answers

a) $3y = -\frac{x}{4} - 1$

$y = -\frac{x}{12} - \frac{1}{3}$

$\Rightarrow m = -\frac{1}{12}$

new line parallel, so same gradient:

$y = -\frac{1}{12}x + c$

sub in $(0,3)$

$3 = -\frac{1}{12}(0) + c$

$\Rightarrow c = 3$

$y = -\frac{1}{12}x + 3$

b) $2y = -4x + 15$

$y = -2x + \frac{15}{2}$

$\Rightarrow m = -2$

perpendicular,

so $m_2 = \frac{-1}{-2} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

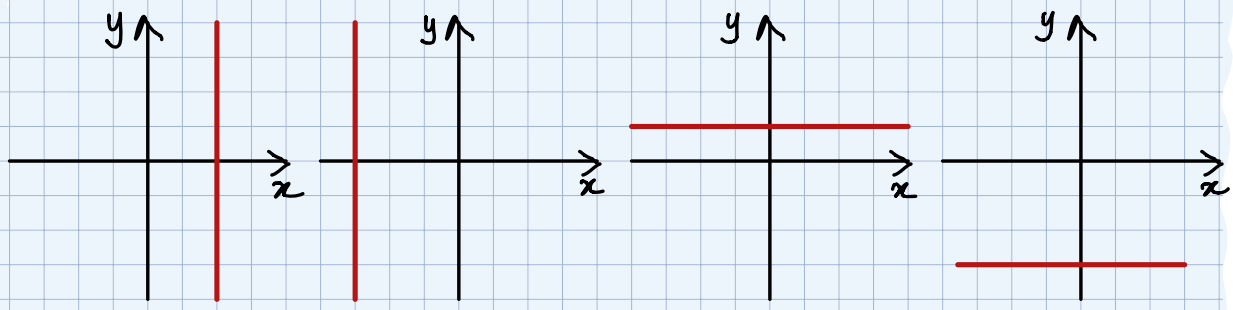
sub in $(1,4)$

$4 = \frac{1}{2}(1) + c$

$\Rightarrow c = 3.5$

$y = \frac{1}{2}x + 3.5$

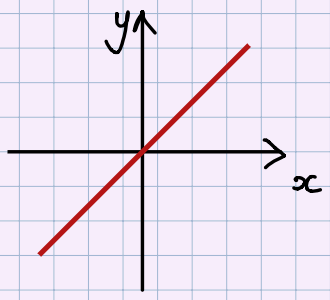
Vertical and Horizontal Lines



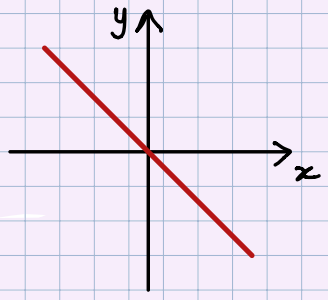
$x = 2$ $x = -3$ $y = 1$ $y = -3$

Hint, all the coordinates on the line have an x-coordinate of 2.

Other graphs



$y = x$



$y = -x$