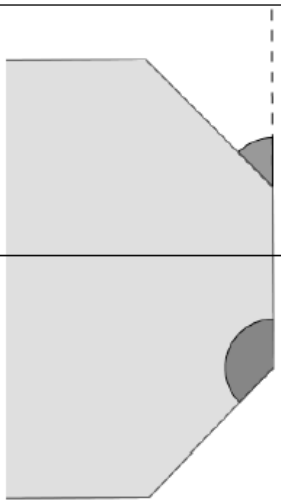
Angles in Polygons

Angles in polygons	**Any** polygon with n sides	A **regular** polygon with n sides
Exterior angles ('angle at centre' follows same rules) 	$\boldsymbol{Total} = 360$	$\boldsymbol{Each\ angle} = \frac{360}{n}$
Interior angles	$\boldsymbol{Total} = 180(n-2)$	$\boldsymbol{Each\ angle} = 180 - \frac{360}{n}$

Can you explain why *the rules above must be true?*

1. Calculate the total of the interior angles in a pentagon. Show your working.

2. What is the interior angle of a regular decagon?

3. What is the angle at the centre of a regular heptagon (7-sided shape)?

4. Write down an expression for the size of the exterior angle for an n-sided shape.

5. The sum of the interior angles of a polygon is 1440°. How many sides does it have?

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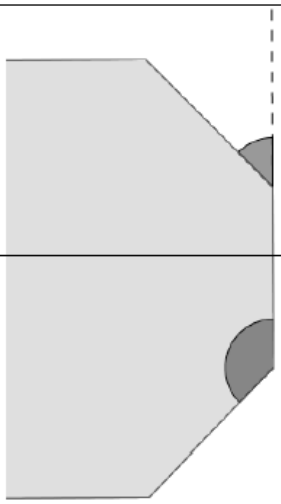
Exterior angles ('angle at centre' follows same rules) 	$\boldsymbol{Total} = 360$	$\boldsymbol{Each\ angle} = \frac{360}{n}$
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Can you explain why *the rules above must be true?*

- Exterior angles add up to $360°$ because if you turn around each exterior angle you must complete a full turn, regardless of the number of sides. The greater the number of sides, the smaller each angle is likely to be.
- In a regular polygon, each angle is equal.
- A polygon with n sides can be split into $n-2$ triangles (with all triangle angles in the corners), and each triangle's angles add up to $180°$.
- The best way to find each interior angle for a regular polygon is to subtract the exterior angle from $180°$.

1. Calculate the total of the interior angles in a pentagon. Show your working.

$$180(n-2) = 180(5-2) = 180 \times 3 = \mathbf{540°}$$

2. What is the interior angle of a regular decagon?

$$\text{Exterior: } \frac{360}{10} = 36°$$

$$\text{Interior: } 180 - 36 = \mathbf{144°}$$

3. What is the angle at the centre of a regular heptagon (7-sided shape)?

$$\frac{360}{7} = 51\frac{3}{7} = \mathbf{51.\dot{4}2857\dot{1}°}$$

4. Write down an expression for the size of the exterior angle for an n-sided shape.

$$\boldsymbol{\frac{360}{n}}$$

5. The sum of the interior angles of a polygon is 1440°. How many sides does it have?

$$180(n-2) = 1440 \implies n - 2 = \frac{1440}{180} = \frac{144}{18} = \frac{72}{9} = 8 \implies \boldsymbol{n = 10}$$